

Cotunneling Transport and Quantum Phase Transitions in Coupled Josephson-Junction Chains with Charge Frustration

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We investigate the quantum phase transitions in two capacitively coupled chains of ultra-small Josephson-junctions, where the particle-hole symmetry is broken by the gate voltage applied to each superconducting island. Near the maximal-frustration line, cotunneling of the particles along the two chains is shown to play a major role in the transport and to drive a quantum phase transition out of the charge-density wave insulator, as the Josephson-coupling energy is increased. We also argue briefly that slightly off the symmetry line, the universality class of the transition remains the same as that right on the line, being driven by the particle-hole pairs.

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Systems of ultra-small tunnel junctions composed of metallic or superconducting electrodes have been the source of a great number of experimental and theoretical works [1]. Single charge (electron or Cooper pair) tunneling observed in those systems demonstrates the remarkable effects of Coulomb blockade. Especially, in Josephson-junction arrays, the charging energy in competition with the Josephson-coupling energy further brings about the noble effects of quantum fluctuations, which induce quantum phase transitions at zero temperature [2]. Very recently, another fascinating manifestation of Coulomb blockade has been revealed in capacitively coupled one-dimensional (1D) arrays of metallic tunnel junctions [3,4]: In such coupled chains, the major transport along both chains occurs via *cotunneling* of the electron-hole pairs, which is a quantum mechanical process through an intermediate virtual state. Such a cotunneling transport leads to the interesting phenomenon of the current mirror.

In capacitively coupled Josephson-junction chains, the counterpart of the electron-hole pair is the particle-hole pair, i.e., the pair of an excess and a deficit in Cooper pairs across the two chains. Such particle-holes pairs, combined with the quantum fluctuations, have been proposed to drive the insulator-to-superconductor transition [5]. Here it should be noticed that the particle-hole pair is stable only near the particle-hole symmetry line; far away from the symmetry line, it does not make the lowest charging-energy configuration anymore. Moreover, in a single chain of Josephson junctions, breaking the particle-hole symmetry (by applying a gate voltage) is known to change immediately the universality class of the transition [6,7]. Therefore, it is necessary to find another relevant cotunneling process, if any, off the particle-hole symmetry line and to examine how the transitions change in coupled Josephson-junction chains.

As an attempt toward that goal, we investigate in this paper the quantum phase transitions in two chains

of ultra-small Josephson-junctions, coupled capacitively with each other. The particle-hole symmetry is broken by the gate voltage applied to each superconducting island; the resulting induced charge introduces *frustration* to the system. Near the maximal-frustration line, cotunneling of the particles along the two chains is found to play a major role in the transport and to drive a quantum phase transition out of the charge-density wave (CDW) insulator, as the Josephson-coupling energy is increased. We also argue that slightly off the symmetry line, the universality class of the transition remains the same as that right on the line, i.e., a Berezinskii-Kosterlitz-Thouless (BKT) transition [8], driven by the particle-hole pairs.

We consider two 1D arrays, i.e., chains of Josephson junctions, each of which is characterized by the Josephson coupling energy E_J and the charging energies $E_0 \equiv e^2/2C_0$ and $E_1 \equiv e^2/2C_1$, associated with the self-capacitance C_0 and the junction capacitance C_1 , respectively (see Fig. 1). The two chains are coupled with each other via the capacitance C_I , with which the electrostatic energy $E_I \equiv e^2/2C_I$ is associated, while no Cooper-pair tunneling is allowed between the two chains [9]. The intra-chain capacitances are assumed to be so small ($E_J \ll E_0, E_1$) that, without the coupling, each chain would be in the insulating phase [10]. We are interested in the limit where the coupling capacitance is sufficiently large compared with the intra-chain capacitances, $C_I \gg C_0, C_1$, i.e., $E_I \ll E_0, E_1$ [see Eq. (4) below]. On each superconducting island, external gate voltage V_g is applied, and accordingly, the external charge $n_g \equiv C_0 V_g / 2e$ is induced, with e being the electric charge. The external charge n_g breaks the particle-hole symmetry of the system, introducing charge frustration. We restrict our discussion to two regions: near the particle-hole symmetry line ($|n_g - \mathcal{N}| \ll 1/4$ with \mathcal{N} integer) and near the maximal-frustration line ($|n_g - \mathcal{N} - 1/2| \ll 1/4$), where the properties of the system are severely different. We further note the invariance with respect to the sub-

stitution $n_g \rightarrow n_g + 1$, and take $\mathcal{N} = 0$ without loss of generality.

$$H = 2e^2 \sum_{\ell, \ell'; x, x'} [n_\ell(x) - n_g] \mathbb{C}_{\ell\ell'}^{-1}(x, x') [n_{\ell'}(x') - n_g] - E_J \sum_{\ell, x} \cos[\phi_\ell(x) - \phi_\ell(x+1)], \quad (1)$$

where the number $n_\ell(x)$ of the Cooper pairs and the phase $\phi_\ell(x)$ of the superconducting order parameter at site x on the ℓ th chain ($\ell = 1, 2$) are quantum-mechanically conjugate variables: $[n_\ell(x), \phi_\ell(x')] = i\delta_{\ell\ell'}\delta_{xx'}$. The capacitance matrix \mathbb{C} in Eq. (1) can be written in the block form:

$$\mathbb{C}_{\ell\ell'}(x, x') \equiv C(x, x') \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta_{x, x'} C_I \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (2)$$

with the intra-chain capacitance matrix

$$C(x, x') \equiv C_0 \delta_{xx'} + C_1 [2\delta_{xx'} - \delta_{x, x'+1} - \delta_{x, x'-1}].$$

For simplicity, we keep only the on-site and the nearest neighbor interactions between the charges (i.e., $C_1/C_0 \lesssim 1$) although this is not essential in the subsequent discussion (as long as the interaction range is finite). With the block form of the capacitance matrix in Eq. (2), the Hamiltonian can be conveniently expressed as the sum

$$H = H_C^0 + H_C^1 + H_J \quad (3)$$

with the components

$$\begin{aligned} H_C^0 &\equiv U_0 \sum_x [n_+(x) - 2n_g]^2 + V_0 \sum_x [n_-(x)]^2 \\ H_C^1 &\equiv U_1 \sum_x [n_+(x) - 2n_g][n_+(x+1) - 2n_g] \\ &\quad + V_1 \sum_x n_-(x)n_-(x+1) \\ H_J &\equiv -E_J \sum_{\ell, x} \cos[\phi_\ell(x) - \phi_\ell(x+1)], \end{aligned} \quad (4)$$

where $n_\pm(x) \equiv n_1(x) \pm n_2(x)$ and the coupling strengths are given by $U_0 \simeq 2E_0$, $U_1 \simeq 4(C_1/C_0)E_0$, $V_0 \simeq E_I$, and $V_1 \simeq (C_1/C_I)E_I$.

The on-site charging energy term of the Hamiltonian H_C^0 in Eq. (4) reveals clearly the crucial difference between the charge configurations in the system near the maximal-frustration line $n_g = 1/2$ and those near the particle-hole symmetry line $n_g = 0$. In the former region ($|n_g - 1/2| \ll 1$), the charge configurations which do not satisfy the condition $n_+(x) = 1$ (for all x) have a huge excitation gap of the order of E_0 . (Note that we are interested in the parameter regime $E_I, E_J \ll E_0, E_1$.) Furthermore, the ground states of H_C^0 , separated from the

The Hamiltonian describing the system is given by

excited states by the gap of the order of E_I , have two-fold degeneracy for each x , corresponding to $n_-(x) = \pm 1$. This degeneracy is lifted as the Josephson-coupling energy E_J is turned on. As a result, it is convenient in this case to work within the reduced Hilbert space \mathcal{E}_d , where $n_+(x) = 0$ and $n_-(x) = \pm 1$ for each x . In the latter region ($|n_g| \ll 1/4$), on the other hand, the low-energy charge configuration should satisfy the condition $n_+(x) = 0$ for all x . Unlike the former case, the ground state of H_C^0 is non-degenerate and forms a Mott insulator characterized by $n_1(x) = n_2(x) = 0$ for all x . As E_J is turned on, the ground state of H_C^0 is mixed with the states with $n_-(x) = \pm 2$. Accordingly, the relevant reduced Hilbert space is given by \mathcal{E}_s , where $n_+(x) = 0$ and $n_-(x) = 0, \pm 2$ for all x (see also Ref. [5]).

We first consider the region near the maximal-frustration lines, where we project the Hamiltonian Eq. (1) onto \mathcal{E}_d , and analyze the properties of the system near the maximal-frustration line ($|n_g - 1/2| \ll 1/4$), based on the resulting effective Hamiltonian. Given the projection operator P onto \mathcal{E}_d , the effective Hamiltonian up to the second order in E_J/E_0 ,

$$H_{\text{eff}} \equiv P \left[H_{QPM} + H_J \frac{1-P}{E - H_C^0} H_J \right] P, \quad (5)$$

can be obtained via the standard procedure [6,11]. Implementing explicitly the projection procedure, we get the effective Hamiltonian describing a *single* spin-1/2 anti-ferromagnetic Heisenberg chain [12]

$$\begin{aligned} H_{\text{eff}} &= \gamma J \sum_x S^z(x) S^z(x+1) \\ &\quad - \frac{1}{2} J \sum_x [S^+(x) S^-(x+1) + S^-(x) S^+(x+1)], \end{aligned} \quad (6)$$

where the exchange interaction and the uniaxial anisotropy factor are given by $J \equiv E_J^2/4E_0$ and $\gamma \equiv 16\lambda^2 E_I^2/E_J^2$, respectively. The pseudo-spin operators have been defined according to:

$$\begin{aligned} S^z(x) &\equiv P \frac{n_1(x) - n_2(x)}{2} P \\ S^+(x) &\equiv P e^{-i\phi_1(x)} (1-P) e^{+i\phi_2(x)} P \\ S^-(x) &\equiv P e^{-i\phi_2(x)} (1-P) e^{+i\phi_1(x)} P. \end{aligned} \quad (7)$$

(Note the difference from the standard definition.)

The effective Hamiltonian in Eq. (6) includes contributions from several complex processes, back in the charge picture: The first term in Eq. (6), which comes from the projection PH_C^1P , simply describes the nearest-neighbor interaction of the charges in the form $n_-(x)$. On the other hand, the second term in Eq. (6), arising from the second order expansion $PH_J \frac{1-P}{E-H_C^0} H_J P$, describes the *cotunneling* process of two particles on different chains by way of a quantum mechanical virtual state with energy of the order of E_0 . Figure 2 shows schematically a particle at x on one chain and another particle at $x+1$ (or $x-1$) on the other chain hopping at the same time in the opposite direction. This cotunneling process plays the major role in the charge transport along the two chains, and drives the quantum phase transition in the system, as discussed below.

The antiferromagnetic Heisenberg chain described by Eq. (6) has been extensively studied [13], and is known, from the Bethe ansatz solution or the Sine-Gordon theory [14], to exhibit a quantum phase transition at $\gamma = 1$: For $\gamma > 1$, it belongs to the universality class of the Ising chain in the renormalization group (RG) sense, and its ground state displays genuine long-range order in the staggered magnetization, i.e., $\langle (-1)^x S^z(x) S^z(0) \rangle$ approaches a nonzero constant as $x \rightarrow \infty$. This long-range order in the staggered magnetization corresponds to the charge-density wave (CDW) in the charge picture of the original problem. For $\gamma < 1$, on the other hand, the system described by Eq. (6) is equivalent to the quantum XY chain, where the Mermin-Wagner theorem prohibits genuine long-range order. In this case, the system can be mapped to the repulsive Luttinger model, and the transverse component of the magnetization as well as the z -component of the staggered magnetization exhibits

quasi-long-range order. Namely, both $\langle (-1)^x S^z(x) S^z(0) \rangle$ and $\langle S^+(x) S^-(0) \rangle$ decay algebraically with the distance x , and the system in the charge picture displays *both* the diagonal and the off-diagonal quasi-long-range order. This state may be regarded as the counterpart of the supersolid, possessing both the diagonal and off-diagonal (true) long-range order and proposed recently in 2D Josephson-junction arrays [6].

The properties of the repulsive Luttinger liquid phase, with both the diagonal and off-diagonal quasi-long-range order, has been discussed in Ref. [11] for a *single* Josephson-junction chain. It has been suggested that the system is extremely sensitive to impurities [15] and may make another insulator, different in nature from the CDW insulator. In the coupled chains, the repulsive Luttinger liquid phase has another remarkable feature of the current mirror. According to the basic transport mechanism due to the cotunneling process of particles in the two chains, shown in Fig. 2, the current fed through one chain is accompanied by the secondary current in the other chain, with the same magnitude but in the opposite direction. Similar current mirror effects have also been pointed out for $n_g = 0$, where the mechanism is rather different and via the particle-hole pair transport (see below and Ref. [5]).

We now turn to the region near the particle-hole symmetry line. In the reduced Hilbert space \mathcal{E}_s with the condition $n_+(x) = 0$ satisfied, $n_-(x)/2$ can be regarded as the number of particle-hole pairs located at x . The role of such particle-hole pairs can be analyzed by means of the imaginary-time path-integral representation of the partition function and its dual transformation [5]. The Euclidean action, in the current-loop representation [7], then reads

$$\mathcal{S} = \frac{1}{4K} \sum_{\ell\ell', xx', \tau} [n_\ell(x, \tau) - n_g] [2C_I \mathbb{C}_{\ell\ell'}^{-1}(x, x')] [n_\ell(x', \tau) - n_g] + \frac{1}{4K} \sum_{\ell, x, \tau} [J_\ell(x, \tau)]^2, \quad (8)$$

where the (imaginary) time has been rescaled in units of the inverse Josephson plasma frequency $\omega_p^{-1} \equiv \hbar/\sqrt{4E_I E_J}$, the dimensionless coupling constant defined to be $K \equiv \sqrt{E_J/16E_I}$. Here (n_ℓ, J_ℓ) may be viewed as the current in (1+1)-dimensions, satisfying the continuity equation

$$\nabla_\tau n_\ell(x, \tau) + \nabla_x J_\ell(x, \tau) = 0. \quad (9)$$

With the capacitance matrix Eq. (2), it is convenient to decompose the action in Eq. (8) into the sum $\mathcal{S} = \mathcal{S}_+ + \mathcal{S}_-$:

$$\mathcal{S}_+ \simeq \frac{E_0}{4KE_I} \sum_{x, \tau} [n_+(x, \tau) - 2n_g]^2 + \frac{1}{8K} \sum_{x, \tau} [J_+(x, \tau)]^2$$

$$\mathcal{S}_- \simeq \frac{1}{8K} \sum_{x, \tau} [n_-(x, \tau)]^2 + \frac{1}{8K} \sum_{x, \tau} [J_-(x, \tau)]^2 \quad (10)$$

where $J_\pm(x, \tau) \equiv J_1(x, \tau) \pm J_2(x, \tau)$.

The factor E_0/E_I in the component \mathcal{S}_+ , which is enormous in the parameter regime of interest, again implies the condition $n_+(x) = 0$ already mentioned. Further, the continuity equation in Eq. (9) requires $J_+(x)$ to be a constant on the average, which should obviously be zero. Consequently, near the transition point, the system is effectively described by the action \mathcal{S}_- in Eq. (10), which is equivalent to the 2D XY model, and exhibits a BKT transition at $K = K_{BKT} \approx 2/\pi$ [8].

Here the transition, which is between the Mott insulating phase and the superconducting phase, is driven exclusively by the particle-hole pairs represented by the variable $n_-(x)$, whereas n_+ and J_+ merely renormalize the action \mathcal{S}_- and shift slightly the transition point [5]. This shift of the transition point depends on the external charge n_g and may be estimated in the following way: The transition to the superconducting phase occurs when the Josephson-coupling energy E_J also overcomes the Coulomb blockade associated with a particle-hole pair. Since the Coulomb blockade increases with n_g , approximately given by $8E_0n_g^2+4E_I$, the critical value of E_J is concluded to grow from the symmetry-line value $16K_{BKT}^2E_I$ as n_g is increased. It is also stressed that the BKT-type transition survives the gate voltage as long as the induced charge is sufficiently small ($|n_g| \ll 1/4$); this is in sharp contrast to the single-chain case, where breaking the particle-hole symmetry by nonzero n_g immediately alters the universality class of the transition [6].

In the parameter regimes other than those considered above, the behavior of the system may be inferred by the following argument: First, it is obvious that for $E_J \gg E_0$, the system should be a superconductor with each chain superconducting separately. Note that this superconducting phase, denoted by S, comes from the *particle* (Cooper pair) transport as usual, thus different in character from the superconducting phase in the region $E_J \ll E_0$. In the latter, denoted by S', only the coupled chains as a whole is superconducting, with superconductivity arising from the *particle-hole pair* transport. Far away from both the particle-hole symmetry line and the maximal frustration lines, the Hamiltonian may be projected onto the subspace where $n_1(x) = 0, 1$ and $n_2(x) = 0, 1$ for all x , and single-particle processes dominate the transport in the system.

The observations so far are summarized by the phase diagram displayed schematically in Fig. 3. The phase transitions of our main concern are represented by the thick solid lines, separating the CDW from the repulsive Luttinger liquid (LL) and the Mott insulator (MI) from the superconductor (S'); the somewhat speculative boundaries discussed above are depicted by dashed lines. Here it is not clear within our approach whether the boundary between the repulsive Luttinger liquid region and the superconducting region in the phase diagram describes a phase transition or merely a crossover. Furthermore, even in the single-chain case, the properties of the repulsive Luttinger liquid phase is controversial, and the possibility of an intermediate *normal* phase has recently been raised as well [16].

Coupled chain systems can presumably be realized in experiment by current techniques, which have already made it possible to fabricate submicron metallic junction arrays with large inter-array capacitances [4] as well as large arrays of ultra-small Josephson junctions [1]. We also point out that quasiparticles have been safely dis-

regarded in obtaining the equilibrium properties at zero temperature.

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- [1] *Single Charge Tunneling: Coulomb Blockade Phenomena in Nanostructures*, edited by H. Grabert and M. Devoret (Plenum Press, New York, 1992); G. Schön and A. D. Zaikin, Phys. Rep. **198**, 237 (1990).
- [2] *Josephson Junction Arrays*, edited by H. A. Cerdeira and S. R. Shenoy (North-Holland, Amsterdam, 1996) [Physica B **222** (4), 253 (1996)].
- [3] D. V. Averin, A. N. Korotkov, and Y. V. Nazarov, Phys. Rev. Lett. **66**, 2818 (1991).
- [4] M. Matters, J. J. Versluis, and J. E. Mooij, Phys. Rev. Lett. **78**, 2469 (1997).
- [5] M.-S. Choi, M. Y. Choi, and S.-I. Lee, preprint (cond-mat/9802199); M.-S. Choi, preprint (cond-mat/9802237).
- [6] C. Bruder, R. Fazio, and G. Schön, Phys. Rev. B **47**, 342 (1993).
- [7] A. van Otterlo and K.-H. Wagenblast, Phys. Rev. Lett. **72**, 3598 (1994).
- [8] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **59**, 907 (1970), [Sov. Phys. JETP **32**, 493 (1971)]; J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); J. M. Kosterlitz, *ibid.* **7**, 1047 (1974).
- [9] Capacitively coupled Josephson-junction arrays should be distinguished from the Josephson ladder systems with Cooper-pair tunneling allowed between arrays. See, e.g., E. Granato, Phys. Rev. B **42**, 4797 (1990); **45**, 2557 (1992); **48**, 7727 (1993).
- [10] R. M. Bradley and S. Doniach, Phys. Rev. B **30**, 1138 (1984); M.-S. Choi, J. Yi, M. Y. Choi, J. Choi, and S.-I. Lee, *ibid.* **57**, R716 (1998).
- [11] L. I. Glazman and A. I. Larkin, Phys. Rev. Lett. **79**, 3736 (1997).
- [12] The minus sign in front of the second term in Eq. (6) is not crucial here on a biparticle lattice (see Ref. [13]).
- [13] A. Auerbach, *Interacting Electrons and Quantum Magnetism* (Springer-Verlag, Berlin, 1994).
- [14] E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison-Wesley, New York, 1991).
- [15] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
- [16] See, e.g., T. D. Küner and H. Monien, preprint (cond-mat/9712307), 1997; R. Baltin and K.-H. Wagenblast, preprint (cond-mat/9705261).

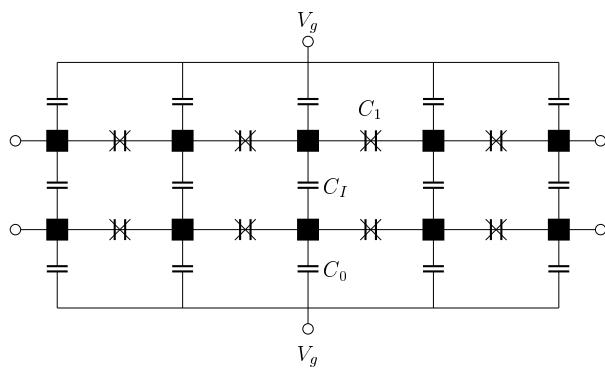


FIG. 1. Schematic diagram of the system.

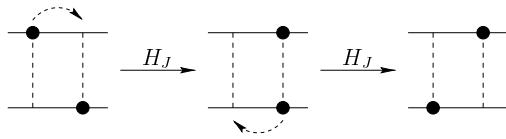


FIG. 2. A typical second-order process via intermediate virtual states with energies of the order of E_0 near the maximal-frustration line.

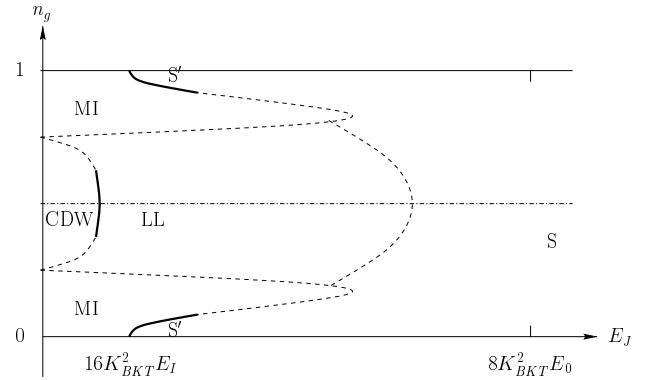


FIG. 3. Schematic phase diagram of the coupled Josephson-junction chains.